TABLE 1. CONCENTRATIONS OF OH^- AND FREE MONOETHANOLAMINE IN SOLUTIONS OF VARIOUS STRENGTHS OF TOTAL AMINE BASED ON DATA OF BATES AND PINCHING

B_0	(B)	(OH^-)	$(B)/(OH^-)$
0.01	0.00945	0.00055	17.3
0.1	0.0982	0.00178	55.1
0.5	0.496	0.00397	125
1.0	0.9944	0.0056	177
2.0	1.9926	0.00744	268

in solutions of various total amine strengths Emmert and Pigford used the basic dissociation constant for this reaction as reported by Bates and Pinching. In the original Bates and Pinching paper, the basic dissociation constant was defined as being equal to K_w, "the ionization constant of water," divided by K_{bh}, the dissociation constant of ethanol ammonium ion. Actually Bates and Pinching intended that K_w be the ion product of water, which is equal to 10^{-14}, instead of the dissociation constant of water which is equal to 1.8×10^{-16}. The consequence is that the equations shown in Column 1, page 173, of Emmert and Pigford's article should read

$$K_b = \frac{(BH^+)(OH^-)}{(B)} \cdot \frac{\gamma BH^+ \gamma OH^-}{\gamma B} = 3.18 \times 10^{-5} \text{ (25°C.)}$$

TABLE 2. RELATIVE INFLUENCE OF PARALLEL, FAST-REACTION BATHS IN CONSUMING CARBON DIOXIDE (BASED ON INITIAL RATES)

B_0	Per Cent of Carbon Dioxide Consumed by *Rx* 1	2	3	Moles amine consumed/mole CO_2 reaction, ν
0.01	91.6	8.1	0.3	1.08
0.1	97.2	2.7	0.1	1.03
0.5	98.6	1.2	0.2	1.01
1.0	98.8	0.9	0.3	1.01
2.0	99.0	0.6	0.4	1.01

and

$$\frac{(BH^+)(OH^-)}{(B)} = 3.18 \times 10^{-5} \text{ (25°C.)}$$

Tables 1 and 2 must then be revised to the following:

So the average stoichiometric factor is estimated theoretically to lie between 1.0 and 1.1, as contrasted with the experimentally observed value of 1.78.

These corrections do not alter Emmert and Pigford's previous conclusions in comparing stoichiometric coefficients extracted from their absorption data with those predicted from mechanism data. The disagreement which previously existed is even greater now. They still have no explanation for this disagreement.

TABLE 3. RATE CONSTANTS FOR REACTION $HOC_2H_4NHCOOH + H_2O$ $HOC_2H_4NH_4CO_3$ CALCULATED FROM DATA OF FAURHOLT (18°C.) (CARBAMATE + H_2O CARBONATE)

B_0	$k_{forward}$, sec.$^{-1}$	$k_{reverse}$, sec.$^{-1}$
1.0	1.2×10^{-6}	1.5×10^{-6}
0.1	1.6×10^{-5}	0.44×10^{-5}
0.01	1.3×10^{-4}	0.12×10^{-4}

The rate constants for the carbamate-to-carbonate reaction calculated from the rate expression of Faurholt, et al. (employing dissociation constants from Bates and Pinching) must also be revised, although little change results. Agreement with the writers' rate measurements for a 0.1N amine solution still remains good. Based on an estimated activation energy Emmert and Pigford's measurement (corrected to 18°C.) gives $k_{forward}$ of 2×10^{-5} sec.$^{-1}$ compared with the calculated value of 1.6×10^{-5} sec.$^{-1}$. Table 3 shows revised values. However the values for the reverse reaction rate constant (which Emmert and Pigford did not employ in interpreting their data) are changed significantly.

COMMUNICATIONS TO THE EDITOR

Equations for Transient Heat Transfer in Packed Beds

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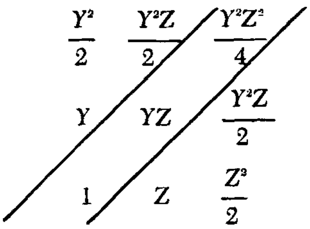
Readers interested in transient heat and mass transfer in packed beds [theories developed since 1926 by Anzelius (*1*), Schumann (*2*), Furnas (*3*), Nusselt (*4*), Thomas (*5*), and many others, see a 1954 survey (*6*)] may have use for the following table, an array of terms from some frequently met infinite double-power series [see (*5*) and (*6*)], higher terms than those of second degree in Y and Z having been omitted only for the sake of simplicity in reproduction:



Starting from unity, in the left-hand bottom corner, each term is derived either from the one below it by an integration with respect to Y or from the one to the left by an integration with respect to Z.

By combining groups of terms with equal degree, one sees that the sum is e^{Y+Z}.

By moving one step in the 45-deg. direction this sum is seen to be a solution of $\frac{\partial^2\phi}{\partial Y \partial Z} = \phi$.

If one divides the table by lines in the 45-deg. direction, as shown, the same differential equation holds true for each part of the table, the boundary conditions varying from case to case because in each section separately all terms are converted into others of the same section by the $\frac{\partial^2}{\partial Y \partial Z}$ operation.

Now if T_1 and T_2 are the solutions of the well-known heat transfer problem

$$\frac{\partial T_2}{\partial Z} = -\frac{\partial T_1}{\partial Y} = T_1 - T_2$$

with

$$Z = 0; \quad T_2 = 0$$
$$Y = 0; \quad T_1 = 1$$

where T_1 and T_2 represent the gas and solid temperatures at place Y and time (corrected for position) Z, the three sections of the table represent (from left to right) see for instance (*6*):

$$e^{Y+Z}(1 - T_1) = e^Y \int_0^Y e^{-u} I_0 \left(2\sqrt{uZ}\right) du = \phi(Y, Z)$$

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(ϕ in Thomas' notation)

$$e^{Y+Z}(T_1 - T_2) = I_0(2\sqrt{YZ})$$

$$e^{Y+Z}T_2 = e^Z \int_0^Z e^{-u} I_0\ (2\sqrt{Yu})\, du = \phi(Z, Y)$$

Some well-known relations in the field (see Thomas) can now be easily proved (by stepping left or down in the diagram) or memorized:

1. e^{Y+Z} = sum of three component parts.

2. $$\frac{\partial \phi(Y, Z)}{\partial Y} = \phi(Y, Z) + I_0(2\sqrt{YZ}).$$

By differentiating the terms of the upper part of the table with respect to Y one gets back those same terms plus those along the diagonal.

3. $$\frac{\partial \phi(Y, Z)}{\partial Z} = \phi(Y, Z) - \frac{\partial I_0(2\sqrt{YZ})}{\partial Z}.$$

4. Similar equations for derivatives of $\phi(Z, Y)$.

5. Limiting cases of $\phi(Y, Z)$ and $\phi(Z, Y)$ for both $Y = 0$ and $Z = 0$.

For numerical use the series expansions are inconvenient because of slow convergence. However since they converge for all Y and Z, they remain suited for mathematical proofs of the above kind.

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A Correlation Between Single Fiber Efficiencies of Fibrous Filters and Operating Variables

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Industrially as well as in laboratories fibrous filters are extensively used for the collection of aerosol particles. Packed beds of glass fibers in particular are being applied to the removal of air-borne microbes in the fermentation industry. For a successful design and maintenance of fibrous filters both values of pressure drop and collection efficiency have to be predicted under any operating condition in practice.

The procedure of calculating the pressure drop of air flow through fibrous beds has been well established (*5*). The effects of various factors, particle size, fiber diameter, air velocity, and so forth, on the collection efficiencies of fibrous filters have also been studied experimentally and theoretically (*1, 2, 4, 6, 7, 8, 9, 10, 11, 12, 13*). However all of the experimental data ever reported on this latter subject remain to be reviewed before reasonable calculations on the collection efficiencies can be made. This situation should have emerged from the fact that the collection efficiencies of fibrous filters are affected not only by the above-mentioned factors but also by the physicochemical interactions which are expected to exist between fibers and particles concerned and which are left open for further discussion and experimentation.

An approach attempted in this paper to correlate between single fiber efficiencies of fibrous beds and operating variables will eventually be of help in clarifying the degree of divergences of data points which have been reported so far and in pointing out a possibility of the existence of other mechanisms, if any, than the inertial impaction, interception, and diffusion of aerosol particles.

If fibrous filters are applied to the removal of air-borne contaminants, for instance bacteria, viruses and so on, they are usually operated satisfying certain conditions under which the deposition of particles due to their inertial impactions are kept minimum. The sizes of bacteria and viruses in question are in a range from about 0.5 to 3.0 μ and from 40 to 80 mμ, respectively. Therefore the following discussions which are exclusively concerned with the interception and diffusion of aerosol particles are not prohibitive in terms of generalization and practical application.

SEMITHEORETICAL CORRELATION BETWEEN SINGLE FIBER EFFICIENCIES AND OPERATING VARIABLES

In accordance with Langmuir (2) a critical value ψ_c of the inertial parameter ψ, below which the deposition of aerosol particles due to their inertial impactions in a single and isolated cylindrical fiber cannot be expected, is as follows:

$$\psi = \psi_c = 1/16 = \frac{C \cdot \rho_p \cdot d_p^2 \cdot v_c}{18\, \mu d_f} \tag{1}$$

From Equation (1)

$$v_c = (1.125) \frac{\mu \cdot d_f}{C \cdot \rho_p \cdot d_p^2} \tag{2}$$

A region, in which the linear velocity of air does not exceed v_c expressed by Equation (2), will be considered tentatively in this paper. Because Equation (2) is based on a potential (nonviscous) flow around cylinders, the real critical velocity may differ from that given by Equation (2) owing to the filtration practice which is usually carried out in the laminar or near laminar region. The determination of the critical velocity is up to pertinent experiments, at any rate (Figure 1).

Also in accordance with Langmuir (2) the collection efficiencies η_o'' and η_o''' for an isolated cylindrical fiber due to the interception and diffusion of spherical particles are given by Equations (3) and (4), respectively:

$$\eta_o'' = \frac{1}{2[2.00 - \ln N_{Re}]} \left[2(1 + N_R) \ln (1 + N_R) - (1 + N_R) + \frac{1}{1 + N_R} \right] \tag{3}$$

where

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